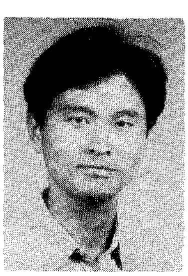
Aeroassisted Orbital Maneuvering Using Lyapunov Optimal Feedback Control

Byoungsoo Lee and Walter J. Grantham
Washington State University, Pullman, Washington

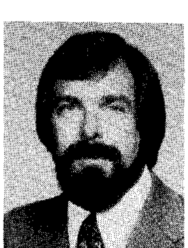
For advanced space transportation systems, such as the National Aerospace Plane or the Orbital Transfer Vehicle, the use of aeroassisted orbital transfer from a high Earth orbit to a low Earth orbit will require an optimizing closed-loop feedback controller, not only to achieve significant fuel savings, but also to compensate for large, unpredictable fluctuations in atmospheric density. We present a Lyapunov optimal function minimizing feedback control algorithm incorporating a preferred direction of motion at each state of the system, opposite to the gradient of a specified "descent function." Lyapunov optimal feedback control is compared to an approximate minimum-fuel calculus of variations open-loop optimal control algorithm based on the 1962 U.S. Standard Atmosphere. The performance of the two controllers is simulated against both the 1962 U.S. Standard Atmosphere and an atmosphere corresponding to the STS-6 Space Shuttle flight, which contains density fluctuations on the order of ±40% compared to the U.S. Standard Atmosphere. In the STS-6 atmosphere the calculus of variations open-loop controller fails to achieve optimal atmospheric exit conditions. In fact, it fails to exit the atmosphere. The Lyapunov optimal feedback controller is able to achieve not only atmospheric exit, despite the ±40% density fluctuations in the STS-6 atmosphere, but also achieves essentially the optimal (minimum-fuel) exit conditions.

Nomenclature

a	= descent function ellipse semimajor axis
b	= descent function ellipse semiminor axis
B	= $\hat{\rho} S h_e \hat{C}_L / 2m$
c	= R/h_e
C_D	= drag coefficient
C_{D_o}	= value of C_D when $C_L = 0$
C_L	= lift coefficient
$\hat{C}_L$	= value of C_L at $(L/D)_{max}$
E	= total energy
h	= altitude
H	= Lyapunov optimization function
HEO	= high Earth orbit
K	= coefficient in parabolic drag polar
L/D	= lift-to-drag ratio
LEO	= low Earth orbit
M	= mass of Earth
m	= vehicle mass
$\mathbf{P}$	= matrix for quadratic descent function
r	= radial distance from Earth's center
r_1	= radius of HEO
r_2	= radius of LEO
r_P	= radius of target perigee
R	= radius of atmosphere
R_E	= radius of Earth
S	= effective vehicle surface area normal to velocity vector
u	= $C_L/\hat{C}_L$ = nondimensional control variable
U	= control constraint set
v	= $V\sqrt{\mu M/R}$
V	= inertial speed
ΔV	= impulsive change in inertial velocity
W	= descent function
x	= state vector = (x_1,x_2,x_3)



Byoungsoo Lee received a B.S. degree in mechanical engineering from Han Yang University, Seoul, Korea, in 1980. After graduation, he was a junior worker at Gold Star Co., Korea, for two years, with an electronic typewriter development team. He has since continued his studies at Washington State University in pursuit of an M.S. degree (received in 1987 in mechanical engineering), and a Ph.D. degree.



Walt Grantham received a B.S. degree (1968) in mechanical engineering from the University of Texas at Arlington, and M.S. (1971) and Ph.D. (1973) degrees in aerospace engineering from the University of Arizona. His research interests are in the areas of nonlinear and optimal control systems and differential games. Dr. Grantham is a co-author of two books and the recipient of two Best Paper awards: the Richard Bellman Award in 1985 at the Fifth International Conference on Mathematical Modeling, and at the 40th Military Operations Research Symposium in 1977. He has been Chairman of a Differential Games session at the AIAA Guidance, Navigation, and Control Conference since 1987 and has presented invited papers at international conferences in New Zealand, the Peoples Republic of China, and Hawaii, and at U.S. national conferences. During the summers of 1985 and 1986 he was a NASA-ASEE Faculty Fellow at NASA Ames Research Center. Dr. Grantham is a Fellow of Sigma Xi, founding president and charter member of Tau Beta Pi (Texas Eta), and a member of Pi Tau Sigma (Texas Alpha-Omega) engineering honor societies. He is also a member of the AIAA and the IEEE (Control Systems Society). Dr. Grantham currently is an associate professor of mechanical engineering in the Department of Mechanical and Materials Engineering at Washington State University.

Received June 2, 1987; revision received Dec. 10, 1987.

x_1 $= h/h_e$
x_2 $= v$
x_3 $= \gamma$
α $= r/R =$ nondimensional radius
β = angle from the descent function gradient vector
γ = flight path angle
δ $= \rho/\hat{\rho}$
μ = gravitational constant
θ = orbital angular position
ρ = atmosphere density
ρ_o = value of ρ at $h = 0$ km
$\hat{\rho}$ = value of ρ at $h = 40$ km
τ $= (t/h_e)\sqrt{\mu M/R}$
ϕ = descent function ellipse rotation angle

Subscripts indicating location

e = at atmospheric entry
f = at atmospheric exit
1 = at *HEO* radius
2 = at *LEO* radius
A = at apogee
P = at perigee

I. Introduction

VARIOUS missions for advanced space transportation systems, such as the Orbital Transfer Vehicle (OTV) and the National Aerospace Plane (NASP), involve transfers from a high Earth orbit (HEO) to a low Earth orbit (LEO). For example, a vehicle might transfer a payload to a geosynchronous orbit and then rendezvous with either a shuttle or a space station, possibly in a different orbit plane. For such missions, significant fuel savings and increased payload capabilities can be achieved by using the Earth's atmosphere for changing the orbit plane and reducing the energy level to achieve a lower orbit. This initial study considers only a planar circular-to-circular HEO to LEO orbit transfer problem.

Aeroassisted orbital maneuvering will be particularly important for the NASP, since the basic requirement that it be able to conduct orbital operations by taking off and landing like a conventional aircraft will severely limit its fuel carrying capacity.

Two principal technical difficulties exist in the use of aeroassisted orbital transfer, either for the OTV or the NASP. One problem is the aerodynamic heating and nonequilibrium aerothermodynamics associated with hypersonic flight in the upper reaches of the atmosphere.[1] The second problem, which is the main focus of this paper, is the design of a guidance algorithm capable of compensating for large unpredictable fluctuations in atmospheric density: so-called "potholes in the sky."[1] To model this uncertainty in density, which can be on the order of $\pm 40\%$, we use atmospheric data derived from the STS-6 Space Shuttle flight.[2]

In order to exploit fully the fuel savings possible with aeroassisted orbital transfers and to handle unpredictable density fluctuations, the guidance algorithm must be a closed-loop feedback controller, computed as a function of the current state of the system and its environment, rather than an open-loop controller based on assumed future conditions along a predicted flight path. For such a closed-loop algorithm we employ a Lyapunov optimal feedback control procedure that we discuss in detail later.

In the past, approximate minimum fuel optimal controls have been developed[3] by solving calculus of variations nonlinear two-point boundary value problems from a particular starting state of the system to a desired final state. This yields an open-loop algorithm based on an assumed atmospheric density model. When a large density fluctuation or other disturbance from nominal conditions is encountered, the open-loop algorithm does not provide any corrective action. To achieve closed-loop control, the calculus of variations two-point boundary value problem would have to be solved either in advance for every possible initial state of the system or in real-time for each current state. Neither approach is feasible for onboard guidance.

In this paper we develop a Lyapunov optimal feedback controller and compare its performance with that of a calculus of variations approximate minimum-fuel open-loop controller developed in Ref. 3. Both controllers are designed based on the 1962 U.S. Standard Atmosphere,[4] but simulation results are developed for both the 1962 U.S. Standard Atmosphere and for the STS-6 atmosphere.

II. Minimum Fuel HEO-to-LEO Orbital Transfer

Consider an idealized minimum-fuel HEO-to-LEO transfer illustrated in Fig. 1. A tangential retro-burn at the HEO radius r_1 could be employed to decrease the velocity by an amount ΔV_1 and inject the vehicle into an elliptical orbit with perigee at the boundary of the atmosphere. When the vehicle arrives at the boundary of the atmosphere at radius R, assume that the vehicle could fly along the edge of the atmosphere until a certain amount of kinetic energy is converted to heat by drag. After sufficient velocity has been depleted, the vehicle would leave the atmosphere boundary tangentially, and a later circularization burn would be employed to achieve LEO. For an absolutely minimum fuel transfer, flight along the edge of the atmosphere would occur at the largest possible radius, i.e., at minimum density. In the limit, the drag would approach zero, and the vehicle would require an infinite number of revolutions to reduce its kinetic energy. This limiting case is not realistic for the additional reason that the vehicle can not generate the lift required to fly along the boundary of the atmosphere.

For a realistic transfer, illustrated in Fig. 2, a larger ΔV_1 is required to ensure sufficient injection into the atmosphere so that the desired kinetic energy reduction occurs before the vehicle exits the atmosphere. The vehicle is constrained to make less than one revolution in the atmosphere and should exit nearly tangentially. Then, with zero lift, the vehicle climbs in an elliptic orbit with apogee at the LEO radius r_2, where the required circular orbit is achieved by a tangential circularizing burn which increases the velocity by an amount ΔV_2.

In addition to a larger ΔV_1 required for a realistic transfer, the final flight path angle γ_f is also larger than the ideal value of zero in order to climb out of the atmosphere. Any γ_f larger than zero leads to a larger ΔV_2 than in the idealized case. The characteristic velocity change $\Delta V_1 + \Delta V_2$ for the idealized orbital transfer produces a lower bound on the fuel required for the HEO-to-LEO orbit change.

A dimensionless expression for the characteristic velocity change in a realistic transfer can be written by defining nondimensional radii

$$\alpha_i = r_i/R$$

and nondimensional velocity changes

$$\Delta v_i = \Delta V_i/\sqrt{\mu M/R}$$

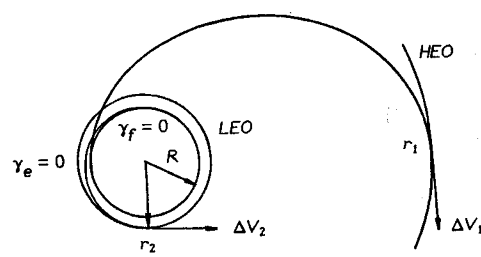


Fig. 1 Idealized orbital transfer.

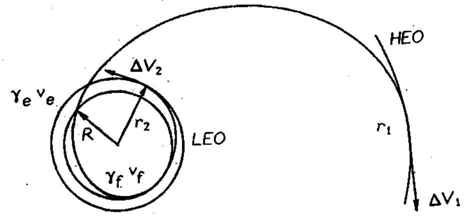


Fig. 2 One-revolution minimum-fuel orbital transfer.

for $i = 1, 2$. As developed in Ref. 5, the dimensionless velocity change for the HEO retroburn is given by

$$\Delta v_1 = \sqrt{1/\alpha_1} - (v_e/\alpha_1)\cos\gamma_e \tag{1}$$

The dimensionless velocity change for the second circularizing burn is

$$\Delta v_2 = \sqrt{1/\alpha_2} - (v_f/\alpha_2)\cos\gamma_f \tag{2}$$

The total characteristic velocity change required for aeroassisted orbital transfer from HEO at radius r_1 to LEO at radius r_2 is

$$\Delta v_1 + \Delta v_2 = \sqrt{1/\alpha_1} - (v_e/\alpha_1)\cos\gamma_e + \sqrt{1/\alpha_2} - (v_f/\alpha_2)\cos\gamma_f \tag{3}$$

These results are derived from the conservation of angular momentum outside the atmosphere. From these relations, the idealized characteristic velocity change, in which both γ_e and γ_f are zero, is given by

$$\Delta v_1 + \Delta v_2 = \sqrt{1/\alpha_1} - (v_e/\alpha_1) + \sqrt{1/\alpha_2} - (v_f/\alpha_2) \tag{4}$$

III. Problem Formulation

The forces acting on the vehicle for planar flight in the atmosphere are illustrated in Fig. 3. For the preliminary analysis in this paper, we assume a Newtonian inverse square gravitational field and a nonrotating atmosphere. The equations of motion are given by

$$\frac{d\theta}{dt} = \frac{V}{r}\cos\gamma \tag{5}$$

$$\frac{dr}{dt} = V\sin\gamma \tag{6}$$

$$\frac{dV}{dt} = -\frac{\rho S C_D V^2}{2m} - \frac{\mu M}{r^2}\sin\gamma \tag{7}$$

$$V\frac{d\gamma}{dt} = \frac{\rho S C_L V^2}{2m} - \left(\frac{\mu M}{r^2} - \frac{V^2}{r}\right)\cos\gamma \tag{8}$$

The lift coefficient C_L, which is modulated by varying the angle of attack, is physically limited by a maximum and minimum value, given by

$$|C_L| \leq C_{L\max}$$

and we assume that the vehicle has a parabolic drag polar

$$C_D = C_{D_o} + KC_L^2 \tag{9}$$

where

$$\hat{C}_L = \sqrt{C_{D_o}/K}$$

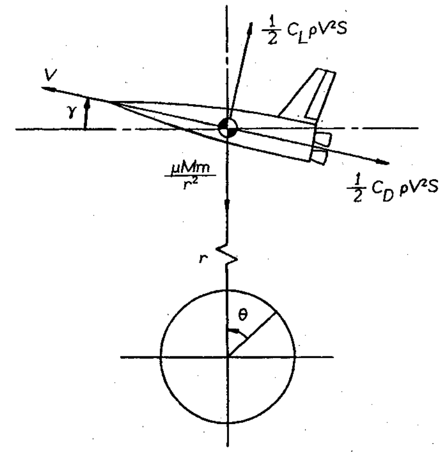


Fig. 3 Forces acting on the vehicle in the atmosphere.

is the lift coefficient at the maximum lift-to-drag ratio

$$\left[\frac{L}{D}\right]_{\max} = \left[\frac{C_L}{C_D}\right]_{\max} = \frac{1}{2\sqrt{KC_{D_o}}}$$

Using the dimensionless state variables:

$$x_1 = h/h_e; \quad x_2 = V/\sqrt{\mu M/R}; \quad x_3 = \gamma \tag{10}$$

and parameters

$$\tau = \frac{t}{h_e}\sqrt{\mu M/R}; \quad \delta = \rho/\hat{\rho}; \quad c = R/h_e; \quad B = \frac{\hat{\rho} S h_e \hat{C}_L}{2m} \tag{11}$$

the equations of motion can be written in terms of the state vector $\boldsymbol{x} = (x_1, x_2, x_3)$ as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\mu}) \tag{12}$$

corresponding to

$$\dot{x}_1 = x_2 \sin x_3 \tag{13}$$

$$\dot{x}_2 = -\frac{B\delta}{2(L/D)_{\max}}(1+u^2)x_2^2 - \frac{c}{(c-1+x_1)^2}\sin x_3 \tag{14}$$

$$\dot{x}_3 = B\delta u x_2 + \frac{\cos x_3}{(c-1+x_1)}\left[x_2 - \frac{c}{(c-1+x_1)x_2}\right] \tag{15}$$

where $(\dot{\ })$ denotes $d(\ )/d\tau$ and u is the lift control, with specified control constraints

$$u \in U = \{u \,|\, |u| \leq u_{\max}\}$$

We do not include Eq. (5) because we are concerned with circular-to-circular orbit transfer and the problem is independent of θ. Equation (5) would be significant for elliptic-to-elliptic orbital transfers.

IV. Calculus of Variations Open-Loop Optimal Control

The minimum-fuel optimization problem is to find a control to minimize the total characteristic velocity, Eq. (3). An equivalent formulation is to maximize

$$J = v_e \cos\gamma_e/\alpha_1 + v_f \cos\gamma_f/\alpha_2 \tag{16}$$

From Ref. 5 the atmospheric entry and exit conditions must satisfy the relations

$$(2 - v_e^2)\alpha_1^2 - 2\alpha_1 + v_e^2 \cos^2\gamma_e = 0 \tag{17}$$

and

$$(2 - v_f^2)\alpha_2^2 - 2\alpha_2 + v_f^2 \cos^2\gamma_f = 0 \tag{18}$$

which are derived from the conservation of energy and angular momentum outside the atmosphere.

The difference between the value ΔV_1 required to target from HEO to a perigee at the boundary of atmosphere and the value ΔV_1 required to target at a perigee at the surface of the Earth is only 13 m/s. However, ΔV_2 is very sensitive to the value of γ_f, such that the value of ΔV_2 required for a given LEO transfer is increased by 100 m/s for each degree above zero for γ_f. Consequently, an approximate minimum-fuel problem can be formulated by only minimizing ΔV_2. The solution to this problem was developed in Ref. 3 and will not be repeated here. However, the resulting open-loop controller was verified and simulated for comparison purposes.

V. Lyapunov Optimal Feedback Control

Knowing that an exit trajectory with $\gamma_f = 0$ leads to the minimum ΔV_2 at the LEO, we employ a Lyapunov optimal controller[6,7] during the atmospheric phase, with the objective of forcing the exit flight path angle γ_f to be near zero but slightly positive, to ensure that the vehicle climbs out of the atmosphere. Our design objective is to drive the system to a final target state, $\hat{\mathbf{x}} = (1, v_f, 0)$, in which $x_3 = 0$ (or as close as possible), x_2 satisfies Eq. (18), and $x_1 = 1$, with less than one revolution in the atmosphere.

A "descent function" $W(\mathbf{x})$ may be thought of as being entirely analogous to "distance" to the target, but, in general, it can be any candidate Lyapunov-type function, i.e., any function such that if $\mathrm{d}W[\mathbf{x}(t)]/\mathrm{d}t < 0$ could be achieved then $x(t)$ would move to the target.

A differentiable scalar function $W(\mathbf{x})$ is a *descent function* if, and only if, the following conditions hold:

i) $W(\mathbf{x}) > 0$ for all $\mathbf{x} \neq \hat{\mathbf{x}}$

ii) $W(\hat{\mathbf{x}}) = 0$

iii) $\partial W(\mathbf{x})/\partial \mathbf{x} \neq 0$ for all $\mathbf{x} \neq \hat{\mathbf{x}}$

A function $u^*(\mathbf{x})$ is a *Lyapunov optimal* feedback control generated by a descent function $W(\mathbf{x})$ if, and only if, for all $\mathbf{x} \neq \hat{\mathbf{x}}$,

$$\mathbf{f}[\mathbf{x}, \mathbf{u}^*(\mathbf{x})] \neq 0$$

and

$$H[\mathbf{x}, \mathbf{u}^*(\mathbf{x})] \leq H(\mathbf{x}, \mathbf{u})$$

for all $\mathbf{u} \in U$ such that $\mathbf{f}(\mathbf{x}, \mathbf{u}) \neq 0$, where

$$H(\mathbf{x}, \mathbf{u}) = \frac{\partial W(\mathbf{x})/\partial \mathbf{x}}{\|\partial W(\mathbf{x})/\partial \mathbf{x}\|} \frac{\mathbf{f}(\mathbf{x}, \mathbf{u})}{\|\mathbf{f}(\mathbf{x}, \mathbf{u})\|} = \cos\beta \tag{19}$$

$\|\cdot\|$ denotes the Euclidian norm, and β is the angle between the gradient vector $\partial W/\partial \mathbf{x}$ and the state space "velocity" vector $\mathbf{f}(\mathbf{x}, \mathbf{u})$, as illustrated in Fig. 4. A Lyapunov control is *proper* if $H[\mathbf{x}, \mathbf{u}^*(\mathbf{x})] < H[\mathbf{x}, \mathbf{u}]$ for all $\mathbf{x} \neq \hat{\mathbf{x}}$ and for all $\mathbf{u} \in U$ such that $\mathbf{f}(\mathbf{x}, \mathbf{u}) \neq 0$ and $\mathbf{f}(\mathbf{x}, \mathbf{u})/\|\mathbf{f}(\mathbf{x}, \mathbf{u})\| \neq \mathbf{f}[\mathbf{x}, \mathbf{u}^*(\mathbf{x})]/\|\mathbf{f}[\mathbf{x}, \mathbf{u}^*(\mathbf{x})]\|$.

For a given descent function $W(\mathbf{x})$, a closed-loop feedback control $\mathbf{u}(\mathbf{x})$ is computed directly by solving the function minimization problem of minimizing $H(\mathbf{x}, \mathbf{u})$ subject to the control constraint $u \in U$. For a "proper" descent function, the minimum produces a unique direction of motion at each state $\mathbf{x}$.

The Lyapunov optimal closed-loop controller tries, in a "steepest descent" fashion, to move the system in a preferred direction at each state, opposite to the gradient of the descent function, or as close to this direction as possible given the dynamics of the system, the constraints on the control, and the actual current value of the atmospheric density, which we assume can be measured.

For the case of aeroassisted orbital transfer, we chose the descent function to be an ellipsoid such that the vehicle initially flies with positive lift, to recover from its plunge into the atmosphere, and then flies at negative lift, in such a way that the vehicle climbs out of the atmosphere and exits with near zero positive flight path angle and the desired exit speed.

The preferred direction of motion near the target state is illustrated in Fig. 5 for the two state variables x_1 and x_3.

Using the parameters in Fig. 5, the equation of the descent function chosen for the HEO-to-LEO aeroassisted orbital transfer problem is given by

$$W(\mathbf{x}) = [x_1 - 1 \; x_3] \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_3 \end{bmatrix} + (x_2 - v_f)^2 \tag{20}$$

where

$$p_{11} = a^2 \sin^2\phi + b^2 \cos^2\phi$$

$$p_{12} = (b^2 - a^2) \sin\phi \cos\phi$$

$$p_{22} = b^2 \sin^2\phi + a^2 \cos^2\phi$$

and the target speed is

$$v_f = \sqrt{2\alpha_2(\alpha_2 - 1)/(\alpha_2^2 - \cos^2 x_3)}$$

The (x_1, x_3) contours are elliptical, with semimajor axis a and semiminor axis b. The ellipses are rotated clockwise by the

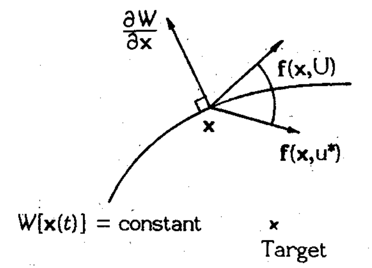


Fig. 4 Gradient of the Lyapunov function and the velocity vector.

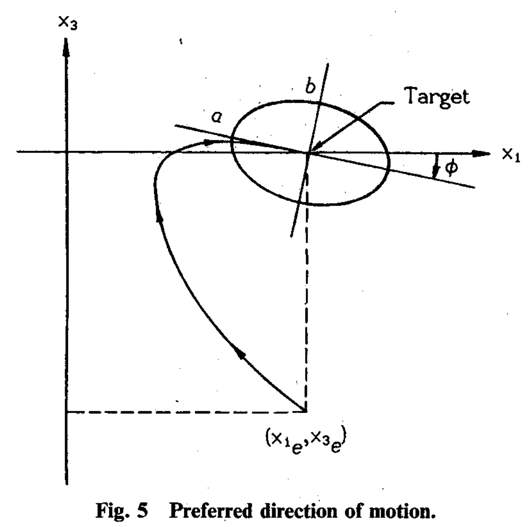


Fig. 5 Preferred direction of motion.

angle ϕ in the $x_1 - x_3$ plane. This descent function corresponds to a preferred direction of approach to the target along the semimajor axis illustrated in Fig. 5. That is, we want to approach the $\gamma_f \cong 0$ flight-path condition at exit, but from a slightly positive flight-path angle, so that the vehicle is climbing out of the atmosphere.

VI. Simulation Results

Using the descent function $W(\mathbf{x})$ given by Eq. (20) and the corresponding minimization function $H[\mathbf{x},\mathbf{u}]$, the lift control to drive the system to the target is derived by minimizing $H[\mathbf{x},\mathbf{u}]$.

In order to minimize the H function, for the case where $|u| < u_{max}$, we differentiate H with respect to u and set the result to zero: $\partial H/\partial u = 0$. As shown in Ref. 5, this yields a fourth-order polynomial in u, with coefficients that are functions of the state variables $\mathbf{x}$. Solving this polynomial, we get the control which minimizes H as one of four roots or as one of the boundary values $\pm u_{max}$. For the root finding scheme, we employed Newton-Bairstow and Quotient-Difference (QD) algorithms,[8] in which QD gives a rough initial value used in the Newton-Bairstow method, with the local absolute error controlled to less than 1.0×10^{-10}.

As discussed in Ref. 5 a target perigee, r_p, is chosen as in Ref. 3, with entry conditions (at $\tau_e = 0$):

$$x_{1_e} = 1 \tag{21}$$

$$x_{2_e} = \sqrt{2[1 - 1/(\alpha_1 + \alpha_p)]} \tag{22}$$

$$x_{3_e} = -\cos^{-1}\sqrt{[2\alpha_1 - (2 - v_e^2)\alpha_1^2]/v_e} \tag{23}$$

and exit conditions (τ_f unspecified):

$$x_{1f} = 1 \tag{24}$$

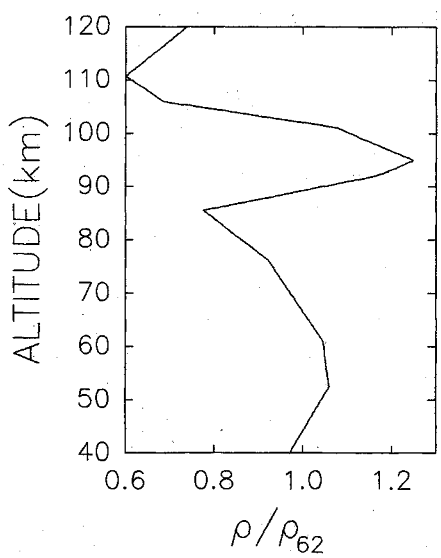


Fig. 6 STS-6 atmospheric density.

Using the initial conditions at entry, along with a computed optimal lift control (either calculus of variations[3] or Lyapunov optimal), Eqs. (13–15) are integrated using a fourth-order Runge-Kutta method. The parameters used for the descent function are

$$a = 40, \qquad b = 2, \qquad \phi = 0.4 \text{ deg}$$

For comparison with the results from Ref. 3, the following vehicle and orbit parameters were used:

$$C_{D_o} = 0.10; \qquad K = 1.11$$

$$|C_L| \le 0.9$$

$$m/S = 300 \text{ kg/m}^2$$

$$r_1 = 42{,}241 \text{ km}; \qquad r_2 = 6578.7 \text{ km}$$

$$r_p = 6400 \text{ km}; \qquad R = 6498 \text{ km}$$

For these parameter values, the nondimensional lift constraint is $|u| \le u_{max} = 2.9985$.

The density of the atmosphere was approximated by a fourteenth-order Chebyshev polynomial whose coefficients were determined by a least-squares fit to the 1962 U.S. Standard Atmosphere (US-62).[4]

Both the Lyapunov optimal feedback controller and the calculus of variations approximate minimum-fuel controller were

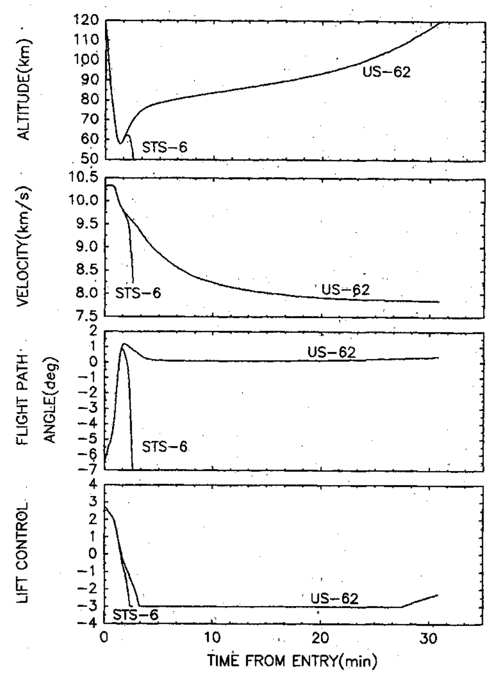


Fig. 7 Time history of the calculus of variations open-loop control state variables and lift control.

Table 1 The characteristic velocity changes and exit conditions

		Calculus of variations		Lyapunov optimal	
	Idealized	US-62	STS-6	US-62	STS-6
v_f, km/s	7.856	7.849	—	7.856	7.875
γ_f, deg	0	0.4	—	0.4297	0.5481
r_2, km	6578.7	6578.7	—	6600.9	6663.6
ΔV_1, m/s	1485.626	1496.048	—	1496.048	1496.048
ΔV_2, m/s	24.056	31.0	—	37.619	56.021
$\Delta V_1 + \Delta V_2$, m/s	1504.154	1527.048	—	1533.668	1552.069

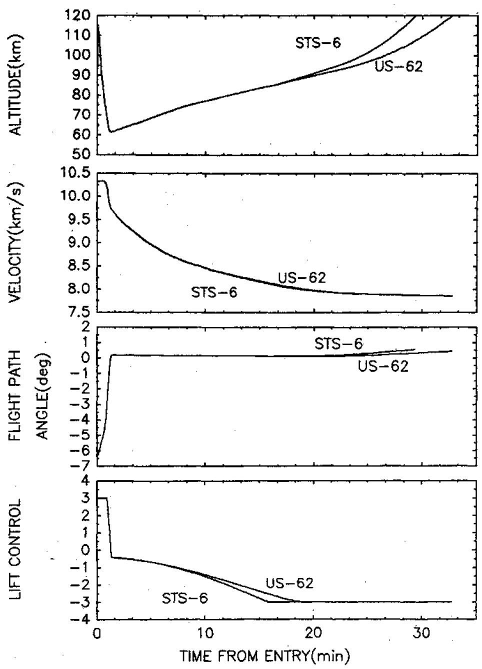


Fig. 8 Time history of the Lyapunov optimal closed-loop control state variables and lift control.

tested against the US-62 atmosphere and against a fluctuated atmosphere derived from the STS-6 flight of the Space Shuttle, illustrated in Fig. 6. The calculus of variations controller does not provide any corrective action against the density fluctuations. It yields an open-loop algorithm based on the assumed US-62 atmospheric density model. This open-loop controller was implemented with fixed parameters, corresponding to flight in the US-62 standard atmosphere. The Lyapunov optimal feedback controller was designed using the US-62 atmosphere, but it was implemented using the actual density.

Time histories of the state variables and the lift controls are plotted in Figs. 7 and 8 for both the US-62 atmosphere and the STS-6 atmosphere. The simulation results for the calculus of variations open-loop controller are shown in Fig. 7. The Lyapunov optimal feedback controller results are shown in Fig. 8. The characteristic velocity changes and exit conditions for the idealized transfer and for the calculus of variations and Lyapunov optimal controllers are listed in Table 1.

VII. Discussion

The calculus of variations open-loop controller, operating in the US-62 atmosphere for which it was designed, achieved an exit flight path angle of $\gamma_f = 0.4$ deg at a speed of $v_f = 7.849$ km/s. These conditions, and the total characteristic velocity change of 1527.048 m/s required for the orbital transfer are very close to the idealized values listed in Table 1.

As indicated in Fig. 7, in the STS-6 atmosphere, the calculus of variations open-loop controller was unable to counteract the large density variations. The vehicle failed to recover from its plunge into the atmosphere and failed to exit the atmosphere.

The Lyapunov optimal feedback controller was designed based on the US-62 atmosphere and a preliminary choice of a descent function but without having to solve any calculus of variations two-point boundary value problems. In the US-62 atmosphere the Lyapunov optimal controller achieved an exit flight path angle of $\gamma_f = 0.4297$ deg at a speed of $v_f = 7.856$ km/s. The ΔV_2 value of 37.619 m/s, applied at apogee in the exit-to-LEO transfer ellipse, yields a higher LEO radius $r_2 = 6600.9$ km than the target radius of 6578.7 km. The resulting total characteristic velocity change of 1533.668 m/s is also higher than for the US-62 optimal calculus of variations controller.

For the STS-6 atmosphere, the Lyapunov controller was able to achieve a LEO, but at a somewhat higher radius $r_2 = 6663.6$ km than desired.

It should be noted that the Lyapunov optimal controller is only a preliminary design and has only been simulated against two possible atmospheres, although the atmospheric density varied on the order of $\pm 40\%$ between the two test cases. Further research is needed to design a game theoretic Lyapunov optimal controller to handle a "worst case" atmosphere, where the magnitude of the density variation can be anything within specified bounds.

VIII. Conclusion

In the STS-6 atmosphere, the calculus of variations open-loop controller was unable to counteract the density fluctuations and the vehicle failed to exit the atmosphere. The Lyapunov optimal feedback controller, which was designed based on the US-62 standard atmosphere but without having to solve any calculus of variations two point boundary value problem, could achieve not only optimal exit conditions but also a low Earth orbit, although at a somewhat higher radius than desired. The Lyapunov optimal feedback controller is a good candidate not only for a feedback guidance law but also for onboard computation, since function minimization is much easier to perform than calculus of variations.

Acknowledgments

Funds for the support of this study have been allocated by the NASA-Ames Research Center, Moffet Field, CA, under interchange No. NCA 2-191.

References

[1]Talay, T. A., White, N. H., and Naftel, J. C., "Impact of Atmospheric Uncertainties and Real Gas Effects on the Performance of Aero-Assisted Orbital Transfer Vehicles," *Proceedings of the AIAA Aerospace Science Meeting*, AIAA, New York, 1984.

[2]Findley, J. T. and McConnell, J. G., "Atmospheres for Aeroassisted Orbital Transfer Vehicles Based on Shuttle Flight Experience," Analytical Mechanics Associates, Hampton, VA, AIAA Paper 83-18, Sept. 1983.

[3]Mease, K. D. and Vinh, N. X., "Minimum-Fuel Aeroassisted Coplanar Orbit Transfer Using Lift-Modulation," *Journal of Guidance, Control, and Dynamics,* Vol. 8, Jan.–Feb. 1985, pp. 134–141.

[4]*U.S. Standard Atmosphere Supplements 1966,* U.S. Govt. Printing Office, Washington, DC, 1966, pp. 67–69.

[5]Lee, B., "Aeroassisted Orbital Maneuvering Using Lyapunov Optimal Feedback Control," M.S. Thesis, Dept. of Mechanical and Materials Engineering, Washington State Univ., Pullman, WA, April 1987.

[6]Grantham, W. J. and Chingcuanco, A. O., "Lyapunov Steepest Descent Control of Constrained Linear Systems," *IEEE Transactions on Automatic Control*, Vol. AC-29, Aug. 1984, pp. 740–743.

[7]Grantham, W. J., "ODESSYS: ODE Simulation SYStem for Nonlinear Optimization, Controls and Differential Games," *Proceedings of the 3rd IEEE Computer Aided Control Systems Design Conference*, Arlington, VA, Sept. 1986.

[8]Ralston, A. and Wilf, H. S., *Mathematical Methods for Digital Computers*, 2nd ed., Wiley, New York, 1967, pp. 37–54.